The Screening Effectiveness of Locating Arrays

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A Combinatorial Testing Model

- Given k factors $F_1, \ldots, F_k$.
- Each factor F_i has a set $S_i = \{v_{i1}, \ldots, v_{is_i}\}$ of s_i possible levels.
- A test is an assignment, for each $i = 1, \ldots, k$, of a level from $\{v_{i1}, \ldots, v_{is_i}\}$ to F_i.

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- The execution of a test yields a measurement of a response.

- The objective of a screening experiment is to identify the factors – and/or interactions – that significantly affect a response.
  - When $\{i_1, \ldots, i_t\} \subseteq \{1, \ldots, k\}$ and $v_{i_j} \in S_{i_j}$, the set $\{(F_{i_j}, v_{i_j}) : 1 \leq j \leq t\}$ is a t-way interaction.

Covering Arrays

A covering array of strength t is an $N \times k$ array of type $(s_1, \ldots, s_k)$ in which, for every $N \times t$ subarray, each level-wise t-way interaction is covered, i.e., occurs, in at least one run.

Example: A 9×4 covering array of strength $t = 2$ of type $(2, 2, 3, 3)$.

Test	A	B	C	D
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
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Why should we care about coverage in a screening design?
If we don't test a factor-level combination, we can't determine if it affects a response significantly.

Coverage is not Enough for Location!

While a CA of strength t covers all t-way interactions, it does not ensure that it is possible to distinguish the influence of different t-way interactions — important for the analysis of experimental results.

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(Not a unique example in this array.)

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Locating arrays strengthen covering arrays to address this very issue.

Locating Arrays

A (d, t)-locating array (LA) is a CA of strength t with an added locating property:

- Any set of d, t-way interactions can be distinguished from any other such set by appearing in a distinct set of tests.

Similar to CAs, LAs scale well with large numbers of factors, k.

- Indeed when the strength t, number d of potentially significant interactions, and maximum number of levels v are fixed, the number of tests required is $O(\log k)$!

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$\Rightarrow$ Locating arrays are efficient screening designs!

Example Locating Array

The CA is not (1, 2)-locating. Each interaction the set $\{A_0B_1, A_0C_2, C_2D_1\}$ only occurs in row 9.

In the (1, 2)-locating array, A_0B_1 occurs in rows $\{4, 5, 6, 7\}$.

Covering Array

Test	A	B	C	D
1	0	0	0	0
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3	0	0	1	0
4	0	0	1	2
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9	1	0	1	1
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11	1	1	2	0
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13	1	1	1	2

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In the $(1,2)$-locating array, A_0C_2 occurs in rows $\{4,5\}$.

Covering Array

Test	A	B	C	D
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	2
5	1	0	2	2
6	1	1	0	2
7	1	1	1	1
8	1	1	2	0
9	0	1	2	1

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In the $(1,2)$-locating array, C_2D_1 only occurs in row $\{4\}$.

But, there is a distinct test to disambiguate each interaction: A_0B_1 occurs in rows $\{4,5,6,7\}$, A_0C_2 occurs in $\{4,5\}$, and C_2D_1 in $\{4\}$.

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Test	A	B	C	D
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	2
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Screening Analysis

For analysis, statisticians prefer experimental designs to be balanced, i.e., have an equal number of observations for all possible level combinations.

- Balance equalizes the variance for each measured factor or interaction.
- But balanced experimental designs are large, e.g., full- or fractional-factorial designs, orthogonal arrays, etc.

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Locating arrays:

- Need not be balanced; there is a trade-off between balance and array size.
- Easily handle categorial factors with mixed-levels in the design.
- Naturally incorporates coverage.

There is little known about the analysis of unbalanced arrays.

Screening: Analysis of Response Data from LAs

Assume a $(1, 2)$-LA is used as the experimental design.

[1] Y. Akhtar, F. Zhang, C. J. Colbourn, J. Stufken & V. R. Syrotiuk (July 2023): Scalable level-wise screening experiments using locating arrays, Journal of Quality Technology, DOI: 10.1080/00224065.2023.2220973

Screening: Analysis of Response Data from LAs

Assume a $(1, 2)$-LA is used as the experimental design.

Now, use a heavy hitters approach to build a model[1]:

- Initialize the model to mean response; obtain initial residuals.
- Repeat until stopping criterion met:
  - Expand the fitted model with the “heaviest” effect (using orthogonal matching pursuit).
  - Update the coefficient estimates (using ordinary least squares).
  - Update the residuals.
  - Score the effect (increment in R^2, adjusted R^2).
- Select significant effects; apply a cutoff.

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The Compressive Sensing Matrix for OMP

Use a greedy selection strategy: Select the "heaviest" effect.

Define a compressive sensing matrix (CSM) for an $N \times k$ locating array A. It has as many rows as runs in A, and columns corresponding to the candidate level-wise terms (plus an intercept, I).

Compressive Sensing Matrix

Locating Array				I	*A*		*B*		*C*			*D*			*AB*				*AC*						*AD*						*BC*						*BD*						*CD*								
A	*B*	*C*	*D*		0	1	0	1	0	1	2	0	1	2	0	0	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	2	2	2
															0	1	0	1	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
0	0	0	0	+	+	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-
0	0	1	1	+	+	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-
0	0	0	2	+	+	-	+	-	+	-	-	-	-	+	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-
0	1	2	1	+	+	-	-	+	-	-	+	-	+	-	-	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-
0	1	2	2	+	+	-	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+
0	1	1	0	+	+	-	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-
0	1	1	1	+	+	-	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-
1	0	2	2	+	-	+	+	-	-	-	+	-	-	+	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	+
1	0	1	1	+	-	+	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-
1	0	0	1	+	-	+	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-
1	1	2	0	+	-	+	-	+	-	-	+	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-
1	1	0	0	+	-	+	-	+	+	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	-	-	-
1	1	1	2	+	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-

The Compressive Sensing Matrix (CSM)

The CSM $M = (m_{ij})$ has $m_{ij} = +1$ if level-wise effect j is covered in the ith run of A, and $m_{ij} = -1$ otherwise.

Example main effect $\{(A, 0)\}$.

Locating Array / Compressive Sensing Matrix

A	B	C	D	I	A		B		C			D			AB				AC						AD						BC						BD						CD								
					0	1	0	1	0	1	2	0	1	2	0	0	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	2	2	2
															0	1	0	1	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
0	0	0	0	+	+	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-
0	0	1	1	+	+	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-
0	0	0	2	+	+	-	+	-	+	-	-	-	-	+	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-
0	1	2	1	+	+	-	-	+	-	-	+	-	+	-	-	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-
0	1	2	2	+	+	-	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+
0	1	1	0	+	+	-	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-
0	1	1	1	+	+	-	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-
1	0	2	2	+	-	+	+	-	-	-	+	-	-	+	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	+
1	0	1	1	+	-	+	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-
1	0	0	1	+	-	+	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-
1	1	2	0	+	-	+	-	+	-	-	+	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-
1	1	0	0	+	-	+	-	+	+	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	-	-	-
1	1	1	2	+	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-

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Example two-way interaction $\{(B, 0), (D, 2)\}$.

Compressive Sensing Matrix

Locating Array				I	A		B		C			D			AB				AC						AD						BC						BD						CD								
A	B	*C*	D		0	1	0	1	0	1	2	0	1	2	0	0	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	2	2	2
															0	1	0	1	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
0	0	0	0	+	+	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-
0	0	1	1	+	+	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-
0	0	0	2	+	+	-	+	-	+	-	-	-	-	+	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-
0	1	2	1	+	+	-	-	+	-	-	+	-	+	-	-	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-
0	1	2	2	+	+	-	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+
0	1	1	0	+	+	-	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-
0	1	1	1	+	+	-	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-
1	0	2	2	+	-	+	+	-	-	-	+	-	-	+	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	+
1	0	1	1	+	-	+	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-
1	0	0	1	+	-	+	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-
1	1	2	0	+	-	+	-	+	-	-	+	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-
1	1	0	0	+	-	+	-	+	+	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	-	-	-
1	1	1	2	+	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-

Using the CSM for OMP

Our greedy selection strategy: Select the “heaviest” effect.

Compressive Sensing Matrix

I	*A*		*B*		*C*			*D*			*AB*				*AC*						*AD*						*BC*						*BD*						*CD*								
	0	1	0	1	0	1	2	0	1	2	0	0	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	2	2	2
											0	1	0	1	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
+	+	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-
+	+	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-
+	+	-	+	-	+	-	-	-	-	+	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-
+	+	-	-	+	-	-	+	-	+	-	-	+	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-
+	+	-	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+
+	+	-	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-
+	+	-	-	+	-	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-
+	-	+	+	-	-	-	+	-	-	+	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	+
+	-	+	+	-	-	+	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-	-	-
+	-	+	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-
+	-	+	-	+	-	-	+	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	-	-	-	+	-	-
+	-	+	-	+	+	-	-	+	-	-	-	-	-	+	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-	-	-	+	-	-	+	-	-	-	-	-	-	-	-
+	-	+	-	+	-	+	-	-	-	+	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	+	-	-	-	-	-	-	+	-	-	-	-	-	+	-	-	-

Residual vector^T

The “heaviest” effect is given by:

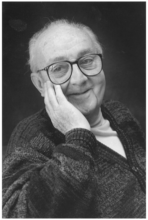
$$\arg\max_i \left| M_i \cdot residuals^T \right|$$

Is the Model Recovered Correct?

While the models produced are not intended to be predictive, they tend to be quite good.

However,

- we have no theorems that we're building the right model, ☹
- so the effects identified may also not be correct. ☹☹

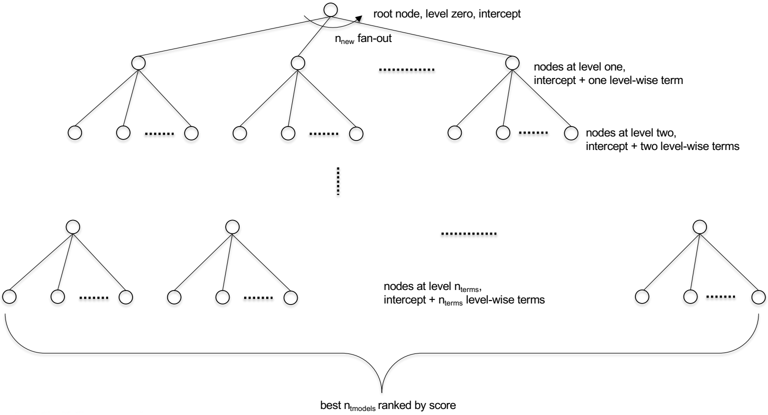


George E.P. Box

"Essentially, all models are wrong, but some are useful."

Use BFS to Generate Many Models

- Rather than generate one model, we generate many models in parallel using breadth-first search (BFS).
  - Aggregate effect scores across models.



Validation of Screening Method

For validation of our screening method, we apply it to several widely studied real data sets using conventional screening designs.

1. A chemical reactor experiment[2] with 5 binary factors.
2. A cast fatigue experiment[3] with 7 binary factors.
3. A contaminant experiment[4] with 9 binary factors.
4. A rubber making experiment[5] with 24 binary factors.

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[2] G. E. P. Box, J. S. Hunter, and W. G. Hunter, Statistics for experimenters, 2nd ed., John Wiley & Sons, Inc., 2005.

[3] G. B. Hunter, F. S. Hodi, and T. W. Eagar. High cycle fatigue of weld repaired cast Ti-6Al-4V. Metallurgical Transactions A, 13:1589-1594, 1982.

[4] A. Miller and R. R. Sitter. Using the folded-over 12-run Plackett-Burman design to consider interactions. Technometrics, 43(1):44-55, 2001.

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We reproduced all these results.

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What Could Improve the Effectiveness of Recovery?

What makes a LA combined with the proposed method of analysis effective?

Are there combinatorial and/or statistical properties of LAs that contribute to their ability to screen effectively?

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Statistical properties (for main-effect models for binary designs):

- The $E(s^2)$ criterion minimizes the sum of squares of the entries of the information matrix.
- The $\max |s|$-criterion considers correlations between columns of the model matrix, and selects a design that minimizes the maximum absolute correlation.
- The r-rank of a design is an indicator of screening effectiveness. We defined a generalization, (r, i)-rank, for binary designs that considers interaction effects.

What Could Improve the Effectiveness of Recovery?

Combinatorial properties:

- In order to quantify the degree to which t-way interactions can be distinguished in an array, the separation between sets of runs for different sets of t-way interactions is introduced:

  For a positive integer δ, an array A is (d, t, δ)-locating if whenever $\mathcal{T}_1, \mathcal{T}_2 \subseteq \mathcal{I}_t$, $|\mathcal{T}_1| = d, |\mathcal{T}_2| = d$, we have that $|(\rho_A(\mathcal{T}_1) \cup \rho_A(\mathcal{T}_2)) \setminus (\rho_A(\mathcal{T}_1) \cap \rho_A(\mathcal{T}_2))| \geq \delta$.

- A (d, t, δ)-locating array guarantees that any two sets of d t-way interactions are separated by at least δ runs.
- By definition, a locating array has a separation of at least one.

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- A (d, t, δ)-locating array guarantees that any two sets of d t-way interactions are separated by at least δ runs.
- By definition, a locating array has a separation of at least one.

A locating array with larger δ is more robust, to e.g., outliers or missing data, however there is a trade-off between large δ and small array size.

Conclusions and Open Problems

- The experimental design and its analysis method are tightly coupled.
- Our results using locating arrays as screening designs together with our analysis algorithm appear promising, with models that fit well, have high overlap in effect identification, and use fewer tests!

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- Our results using locating arrays as screening designs together with our analysis algorithm appear promising, with models that fit well, have high overlap in effect identification, and use fewer tests!

Open problems:

- Improve analysis methods for unbalanced arrays.
  - Investigate more combinatorial and statistical properties.
- Our analysis assumed a $(1, 2)$-LA; how to use more general (d, t)-LAs?
- How to choose among equal “heavy hitters”?
- Rather than BFS perhaps a more intelligent search can be performed?

Source: gograph.com

Questions and/or comments?

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